RADIATION TRANSFER IN A REAL SPECTRUM. INTEGRATION WITH RESPECT TO THE FREQUENCY AND ANGLES

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The method of integration with respect to the frequency of the radiation field in systems with complex spectrum is generalized in order to obtain heat-exchange characteristics integrated with respect to the angles.

1. The method developed in [1-3] to obtain radiation intensity field characteristics integrated over the whole spectrum permits the computation of the radiation heat transfer in three-dimensional systems of any geometry with an arbitrary parameter distribution. If the system does not possess a high degree of symmetry, the integration of the intensity or directional divergence fields with respect to the angles should be performed during solution of the gasdynamic problem.

To illustrate the possibilities of the method of partial characteristics, a number of examples of radiation heat-transfer fields was computed in systems modeling combustion chambers, electric arc apparatus, light punch through during laser-beam focusing [4]. The radiation flux $\vec{S}$ and its divergence $\Delta\vec{S}$ were calculated by means of (1) and (2) from [3]. Examples of such systems are shown in Figs. 1A and B(a). The cylindrical volume in Fig. 1A(a), with a cylinder diameter-to-altitude ratio on the order of one, simulates a combustion chamber or a short electrical arc. The spherical volume in Fig. 1B(a) models the domain of a laser spark with beam focusing from the positive x axis side. Isotherms are shown in Figs. 1A, B(a) by continuous curves on the volume sections, and isobars by the dashes. It is seen from Fig. 1B(a) that the high temperature and pressure domain is shifted against the beam. The system dimensions were selected on the order of several centimeters, the temperature and pressure fields are given in the respective ranges T ~ (10-20)$\cdot 10^3$°K, and P ~ 5-10 bar.

The matrices ΔI and ΔSim (see [3]) were computed within the limits of the parameters mentioned for the model spectrum described in [1-3]. The accompanying spherical (r, θ, φ) coordinate system was coupled to the countable points of the volumes. The outer integrals with respect to θ and φ were evaluated in the usual manner, and the inner integral with respect to r — by means of (6) and (14) from [3], where the limits of the beam 0–L were determined automatically by using a special BC ("boundary checking") procedure. The BC procedure contains information about the boundaries of the volume under consideration, and possesses the property of cutting off integration prior to emergence from the volume in order to prevent the selection of "strange" numbers.

Summation of the divergence was performed algebraically, and their components were stored for the fluxes:

$$\nabla\vec{S} = \int_0^{2\pi}\int_0^{\pi} \nabla I(\theta,\ \varphi)\sin\theta\, d\theta d\varphi. \tag{1}$$

$$S_x = \int_0^{2\pi}\int_0^{\pi} I(\theta,\ \varphi)\sin^2\theta\cos\varphi d\theta d\varphi, \tag{2}$$

$$S_y = \int_0^{2\pi}\int_0^{\pi} I(\theta,\ \varphi)\sin^2\theta\sin\varphi d\theta d\varphi, \tag{3}$$

$$S_z = \int_0^{2\pi}\int_0^{\pi} I(\theta,\ \varphi)\sin\theta\cos\theta d\theta d\varphi. \tag{4}$$

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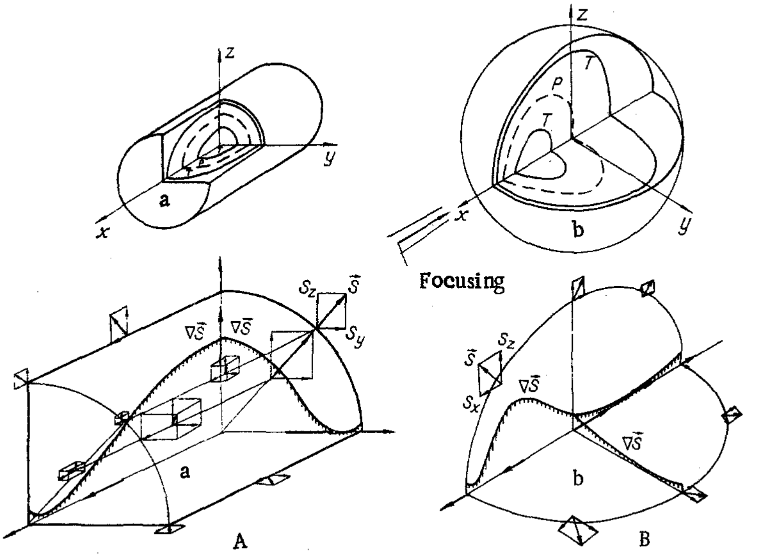

Fig. 1. Radiation flux field and the divergence field in a volume: A) modeling a short electrical arc; B) optical breakdown by a focused laser beam.

Evaluation of the flux $\vec{S}$ and its divergence $\Delta\vec{S}$ at a given point requires just fractions of a second on the BÉSM-6 electronic computer. The results of computations are shown in Figs. 1A and B(b). The flux vectors are shown with their components, and the divergence is displayed by hatched lines.

2. In some cases the gas volume considered in the problem will possess a sufficiently high degree of symmetry and can be replaced approximately by a volume with a one-dimensional parameter distribution. The integration with respect to the angles can hence be performed in advance, just as with respect to the frequency, in the preparation of the partial characteristics. Computation of the flux and divergence fields becomes particularly simple in this case since it is reduced to sampling from an array and a single integration of the appropriate partial characteristics.

The most widespread model of this kind is the approximation of a flat layer [5], which is used extensively in astrophysics and gasdynamics. The computed relationships for the radiation flux and its divergence can be written in this case in the form

$$S(X) = \int_0^L \Delta S(\xi, X)\, \mathrm{sign}(X - \xi)\, d\xi, \tag{5}$$

$$\nabla\vec{S}(X) = \mathrm{So}(X) - \int_0^L \Delta \mathrm{Si}(\xi, X)\, d\xi, \tag{6}$$

$$\Delta S(\xi, X) = 2\int_0^\infty S_\nu^0(\xi)\, k_\nu'(\xi)\, E_2\left(\left|\int_\xi^X k_\nu'(\eta)\, d\eta\right|\right) d\nu, \tag{7}$$

$$\mathrm{So}(X) = 4\int_0^\infty S_\nu^0(X)\, k_\nu'(X)\, d\nu, \tag{8}$$

$$\Delta \mathrm{Si}(\xi, X) = 2\int_0^\infty S_\nu^0(\xi)\, k_\nu'(\xi)\, k_\nu'(X) E_1\left(\left|\int_\xi^X k_\nu'(\eta) d\eta\right|\right) d\nu. \tag{9}$$

The coordinate system here corresponds to Fig. 1 from [3], the notation and meaning of the functionals ΔS, the partial flux ΔSi, the partial sink, as well as the source So are analogous to the intensity introduced for the fields in [3], $E_1(\tau)$ and $E_2(\tau)$ are exponential integrals [5]. Direct utilization of (6), (8), and (9) is inconvenient. In addition to the reasons discussed in [3] and associated with the presence of sections with a high absorption coefficient in the spectrum, additional difficulties are produced by the presence of a zero logarithmic divergence in the function $E_1(\tau)$. Performing an identity transformation analogous to that mentioned in [3], we obtain expressions without the listed inconveniences. Furthermore, introducing splines modeling the distribution of the parameters T and P on the path $x = |\xi - X|$, we will have the final computational scheme for the flat layer

$$\Delta S = 2\int_0^\infty S_\nu^0(T_\xi)\, k_\nu'(T_\xi,\ P_\xi)\, E_2\Big(\int_0^x k_\nu'(\eta)\, d\eta\Big)\, d\nu, \tag{10}$$

$$\mathrm{Som} = 2\int_0^\infty S_\nu^0(T_X)\, k_\nu'(T_X,\ P_X)\, E_2\Big(\int_0^x k_\nu'(\eta)\, d\eta\Big)\, d\nu, \tag{11}$$

$$\Delta\mathrm{Sim} = 2\int_0^\infty [S_\nu^0(T_\xi) - S_\nu^0(T_X)]\, k_\nu'(T_\xi, P_\xi)\, k_\nu'(T_X, P_X)\, E_1\Big(\int_0^x k_\nu'(\eta)\, d\eta\Big)\, d\nu. \tag{12}$$

The flux is computed by means of (5), and the divergence by means of

$$\nabla\vec{S}(X) = \mathrm{Som}\Big|_0^X + \mathrm{Som}\Big|_X^L - \int_0^L \Delta\,\mathrm{Sim}(\xi,\ X)\, d\xi. \tag{13}$$

The layer absorptivity $\int_0^x k_\nu'(\eta)\,d\eta$ models the absorption on the path $\xi\to X$ in (10) and (12) and on the paths $X \to 0$ and $X \to L$ in (11). Moreover, just as for the intensity, the source Som agrees with the partial flux ΔS to the accuracy of the indices so that it is necessary to compute just two matrices beforehand: ΔS and $\Delta\mathrm{Sim}$, where the latter has a volume equal to half the volume of the matrix ΔS because of antisymmetry relative to the inversion $\xi \rightleftarrows X$. Methods of approximating the parameter distribution by model splines are identical to methods discussed in [3]. The properties of the functional (10)-(12) do not differ from the properties of the corresponding functionals for the intensity field (see Figs. 5 and 6 in [3]). Let us note that even these and other functionals are conveniently tabulated in dimensionless form to attenuate their dependence on the source parameters. For instance, the expressions

$$\int_0^\infty S_\nu^0(T_\xi)\, k_\nu'(T_\xi,\ P_\xi)\, d\nu \ \text{and} \int_0^\infty [S_\nu^0(T_\xi) + S_\nu^0(T_X)]\, k_\nu'(T_\xi, P_\xi)\, k_\nu'(T_X, P_x)\, d\nu$$

can be used as normalizing functions for the corresponding functionals. The results of comparing exact computations with those obtained by the method of partial characteristics for a flat layer are analogous to the results of comparing the intensity fields (see Figs. 7, 8, 9 in [3]), and are consequently not presented.

3. In a number of cases one-dimensional models of a sphere (for the computation of explosions [6], laser compression of spherical targets [7, 8], etc.) and of an unbounded cylinder in the axial direction (mainly for the analysis of arc discharges [9-11]) are used.

By computing the energy passing through unit area at the point X, which has been emitted by spherical and cylindrical surfaces of unit thickness and of radius ξ, and taking account of absorption on the path from the source to the point X, we obtain expressions for the partial fluxes (Fig. 2a and b):

For the sphere

$$\Delta S = 2\int_0^\infty S_\nu^0(\xi)\, k_\nu'(\xi)\, \xi^2 \int_0^\pi \frac{(X - \xi\cos\theta)\sin\theta}{\eta^3}\exp(-\tau_\nu^*)\, d\theta d\nu, \tag{14}$$

For the cylinder

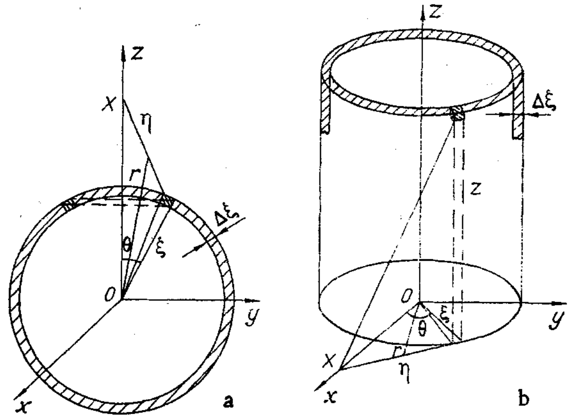

Fig. 2. To derive expressions for the radiation flux in the case of spherical (a) and cylindrical (b) geometries.

$$\Delta S = 2\int_0^\infty S_\nu^0(\xi)\, k_\nu'(\xi)\, \xi \int_0^\pi \frac{2}{\pi}\, \frac{X - \xi\cos\theta}{\eta^2}\, K_0^{(2)}(\tau_\nu^*)\, d\theta d\nu. \tag{15}$$

Here, for generality, the radius of the computed point is denoted by X in both cases, and the polar angle by θ:

$$\eta = \sqrt{X^2 + \xi^2 - 2X\xi\cos\theta}; \tag{16}$$

$$\tau_\nu^* = \int \frac{k_\nu'(r)\, dr}{\sqrt{1 - \left(\frac{X\xi\sin\theta}{r\eta}\right)^2}}. \tag{17}$$

Determination of the function $K_0^{(2)}(\tau_\nu^*)$ and the limits of integration in the integrals with respect to r are discussed below.

The radiation flux at the point X is expressed in terms of the partial quantities (14) or (15) by means of (5), where L should be understood to be the outer radius of the cylinder or sphere in this case.

The expressions for the radiation flux divergence are obtained by differentiating (5) with respect to X in the appropriate coordinate system while taking account of (14) and (15). It is convenient to represent the results of the calculation in the form (6), where So(X) is expressed by means of (8) and

$$\Delta \mathrm{Si}(\xi, X) = 2\int_0^\infty S_\nu^0(\xi)\, k_\nu'(\xi)\, k_\nu'(X)\, \Phi_\nu(X, \xi)\, d\nu. \tag{18}$$

The functionals $\Phi_\nu(X, \xi)$ have the form:

For a sphere

$$\Phi_\nu = \int_0^\pi \left(\frac{\xi}{\eta}\right)^2 \sin\theta \left[1 - F(X, \xi, \theta) + \frac{\xi}{X}\, \frac{2(X^2 + \xi^2)\cos\theta - X\xi(3 + \cos^2\theta)}{\eta^3 k_\nu'(X)}\right] \exp(-\tau_\nu^*)\, d\theta, \tag{19}$$

For a cylinder

$$\Phi_\nu = \frac{2}{\pi}\int_0^\pi \frac{\xi}{\eta}\left\{[1 - F(X, \xi, \theta)]\, K_0^{(1)}(\tau_\nu^*) + \frac{\xi}{X}\, \frac{(X^2 + \xi^2)\cos\theta - 2X\xi}{\eta^3 k_\nu'(X)}\, K_0^{(2)}(\tau_\nu^*)\right\} d\theta. \tag{20}$$

Here

$$F = \frac{X\xi^2 (X\cos\theta - \xi)(X - \xi\cos\theta)\sin^2\theta}{\eta^2 k_\nu'(X)} \int \frac{r k_\nu'(r)\, dr}{(r^2\eta^2 - X^2\xi^2\sin^2\theta)^{3/2}}. \tag{21}$$

The functions $K_0^{(1)}(\tau)$ and $K_0^{(2)}(\tau)$ in (15) and (20) are the first and second integrals of the MacDonald cylinder function $K_0(\tau) = \int_1^\infty \frac{\exp(-\tau x)}{\sqrt{x^2-1}} dx$ with respect to the parameter, respectively;

$$K_0^{(1)}(\tau) = \int_1^\infty \frac{\exp(-\tau x)}{x\sqrt{x^2-1}} dx = \int_\tau^\infty K_0(\alpha)\, d\alpha, \tag{22}$$

$$K_0^{(2)}(\tau) = \int_1^\infty \frac{\exp(-\tau x)}{x^2\sqrt{x^2-1}} dx = \int_\tau^\infty K_0^{(1)}(\alpha)\, d\alpha. \tag{23}$$

The integration parameter x in (22) and (23) is $\sqrt{1+(z/\eta)^2}$. Tables of $K_0^{(1)}(\tau)$ were first calculated in [12], and later reprinted or newly evaluated. Graphs of the function $K_0^{(2)}(\tau)$ are presented in [13].

The limits of integration with respect to r in (17) and (21) can be written in the form of the following scheme:

$$\begin{aligned} &\text{for } \xi \leqslant X, \text{ if } \theta \leqslant \arccos\frac{\xi}{X}, \text{ or else } \int = \int_\xi^X, \text{ or else } \int = \int_\xi^X + 2\int_{r_{\min}}^\xi, \\ &\text{for } \xi \geqslant X, \text{ if } \theta \leqslant \arccos\frac{X}{\xi}, \text{ then } \int = \int_X^\xi, \text{ then } \int = \int_X^\xi + 2\int_{r_{\min}}^\xi. \end{aligned} \tag{24}$$

Here

$$r_{\min} = \frac{X\xi\sin\theta}{\eta}. \tag{25}$$

Let us note that the partial sinks $\Delta Si(\xi, X)$ given by (18) do not have so simple a meaning as absorbed energy from the appropriate sources as in the case of intensity or in the flat layer model. The physical meaning of an energy sink has just an integral in the right side of (6).

Formulas (6), (8), (18)-(21) are suitable for evaluation of the radiation flux divergence fields; however, for the reasons discussed above and in [3], a very small step in integration with respect to ξ is required near the point X. In order to avoid this and to have the opportunity to use a gasdynamic mesh in the computations, (6) can be converted by a method analogous to that used in [3]. The results can be represented in the form (13), which is convenient for the computation of the divergence field. Hence

$$\mathrm{Som}\Big|_\alpha^\beta = 2\int_0^\infty S_\nu^0(X)\, k_\nu'(X)\left(1 - \int_\alpha^\beta k_\nu'(\xi)\,\Phi_\nu(X, \xi)\, d\xi\right) d\nu, \tag{26}$$

$$\Delta\,\mathrm{Sim}(X, \xi) = 2\int_0^\infty [S_\nu^0(\xi) - S_\nu^0(X)]\, k_\nu'(X)\, k_\nu'(\xi)\,\Phi_\nu(X, \xi)\, d\nu. \tag{27}$$

As above, the partial characteristics (14), (15), (26), and (27) can be evaluated in advance if the parameter distribution is modeled by splines. The expressions for the partial characteristics hence remain the same, but the spline reference parameters

$$\begin{aligned} &S_\nu^0(\xi) \to S_\nu^0(T_\xi); \qquad S_\nu^0(X) \to S_\nu^0(T_X); \\ &k_\nu'(\xi) \to k_\nu'(T_\xi, P_\xi); \quad k_\nu'(X) \to k_\nu'(T_X, P_X) \end{aligned} \tag{28}$$

must be substituted as parameters for S_ν^0 and k_ν', and the integrals with respect to r in (17) and (21) must be evaluated taking account of giving $k_\nu'(r)$ by using splines.

The central and axial symmetries of the models under consideration result in the fact that now parabolic splines emerge as the simplest two-point splines which approximate the parame-

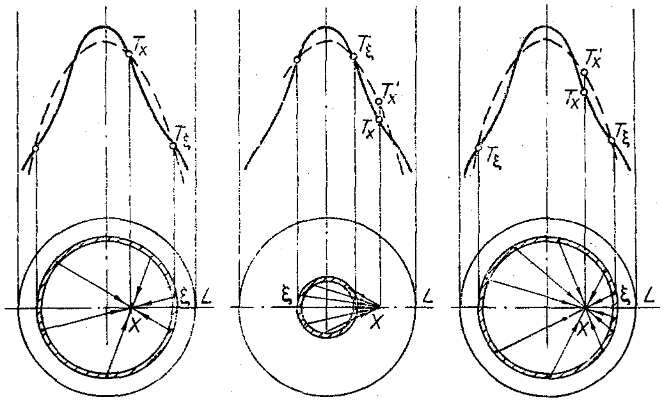

Fig. 3. Approximation of the working temperature distributions by parabolic splines (dashes).

ter distribution in the spherical and cylindrical geometries (Fig. 3) quite well. The equation for the temperature spline relying on the values T_ξ and T_X has the form

$$T(r) = \frac{T_\xi - T_X}{\xi^2 - X^2} r^2 + \frac{\xi^2 T_X - X^2 T_\xi}{\xi^2 - X^2}. \tag{29}$$

The pressure distribution has an analogous form. Parameters of the working profile T_ξ, P_ξ and T_X, P_X (Fig. 3) should be selected as references in modeling the working distribution for the partial sink (27). The best results for the partial fluxes (14) and (15) are obtained when conserving the source parameters T_ξ, P_ξ and selecting the parameters T'_X and P'_X from the condition for conservation of the appropriate quantities. The computational formulas hence have the form (Fig. 3)

$$T'_X = \frac{2X^2}{3\xi^2 - X^2} T_\xi - \frac{3(X^2 - \xi^2)}{X(3\xi^2 - X^2)} \int_0^X T(r)\, dr \tag{30}$$

for $\xi \leqslant X$ and

$$T'_X = \frac{3X^2 - \xi^2}{2\xi^2} T_\xi + \frac{3(\xi^2 - X^2)}{2\xi^3} \int_0^\xi T(r)\, dr \tag{31}$$

for $\xi \geqslant X$.

The presence of absorption rays passing to the computed point from different points of the spatial source ξ (Fig. 3) is taken into account in (30) and (31). The formulas for P'_X are of analogous form. Also, (30) and (31) should be used in approximating the sources (26) by replacing X and T'_X by 0 and T'_0 or L and T'_L, respectively, and ξ and T_ξ by X and T_X for the corresponding terms in (13).

Use of the partial characteristics to compute the heat transfer in the spherical and cylindrical geometries is not more complex than in the plane geometry, however, their preparation is substantially more difficult. This is related to both the more complex form of the expressions to compute the characteristics, and to the large volume of corresponding matrices because of the appearance of two geometric parameters ξ and X in place of one in the case of the intensity field or in the flat layer model. Hence, a computation of the partial fluxes and sinks for the cylindrical and spherical geometries is expedient only when it is necessary to execute mass computations. Otherwise, it is more convenient to use the partial characteristics for the intensity field which permit the solution of the problem in any geometry.

NOTATION

X, x, ξ, η, z, L, r, coordinates of radii in the appropriate formulas; ν, frequency; P, pressure; T, temperature; k'_ν, absorption factor taking forced emission into account; S^o_ν, spectral equilibrium unilateral radiation flux; I, $\vec{S}$, integrated radiation intensity and flux with respect to the frequency; τ, optical density; θ, φ, angles; and α, β, formal parameters.

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QUESTION OF THE NONSTATIONARY RADIANT INTERACTION OF SOLIDS

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The nonstationary thermal interaction by the radiation of an unlimited plate and a source with a constant temperature is considered. A solution is obtained governing the temperature distribution in the plate with any previously assigned accuracy.

The solution of linear-heat-conduction equations with nonlinear boundary conditions is often required in practice. Thus, e.g., at high source temperatures the heat from the source is transmitted to a heated body mainly by radiation. In such cases the convective component turns out to be negligible. Problems of this kind occur every time bodies are heated or cooled in such a way that the convective heat flux is small compared to the radiant heat fluxes.

Despite the fact that the question of heat propagation in solids with radiant heat exchange on the boundaries is encountered quite often as a phenomenon, the number of investigations in the area of nonstationary heat conduction with radiation boundary conditions is, however, quite small [1-8, et al.]. This is evidently explained by the difficulty of the mathematical analysis. Moreover, the present lack of exact solutions of the problems mentioned makes investigation of the regularities of heat propagation in solids subjected to thermal radiation much more difficult.

Meanwhile, the problem of radiant heat transfer becomes more and more valuable in connection with the achievements in studying space and a number of other domains for which large temperature differences are characteristic.

An attempt is made in this paper to find the nonstationary temperature distribution in a plate which initially has the temperature T_0 and is suddenly subjected to the effect of radiation. Mathematically, the problem can be formulated as follows

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